Foundations of Network Diagrams: Dynamical Systems, Bayesian Networks and Quantum Processes

Filippo Bonchi
University of Pisa

Quantum Teleportation

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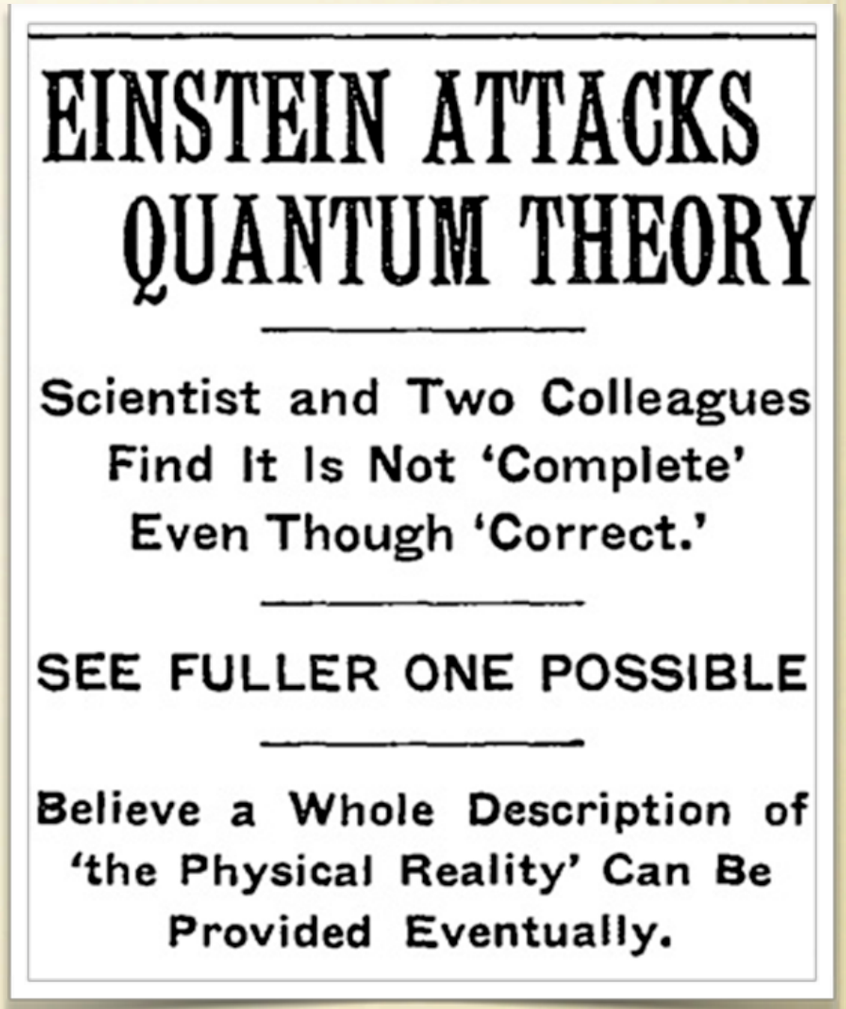

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

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New York Times headline of May 4, 1935.

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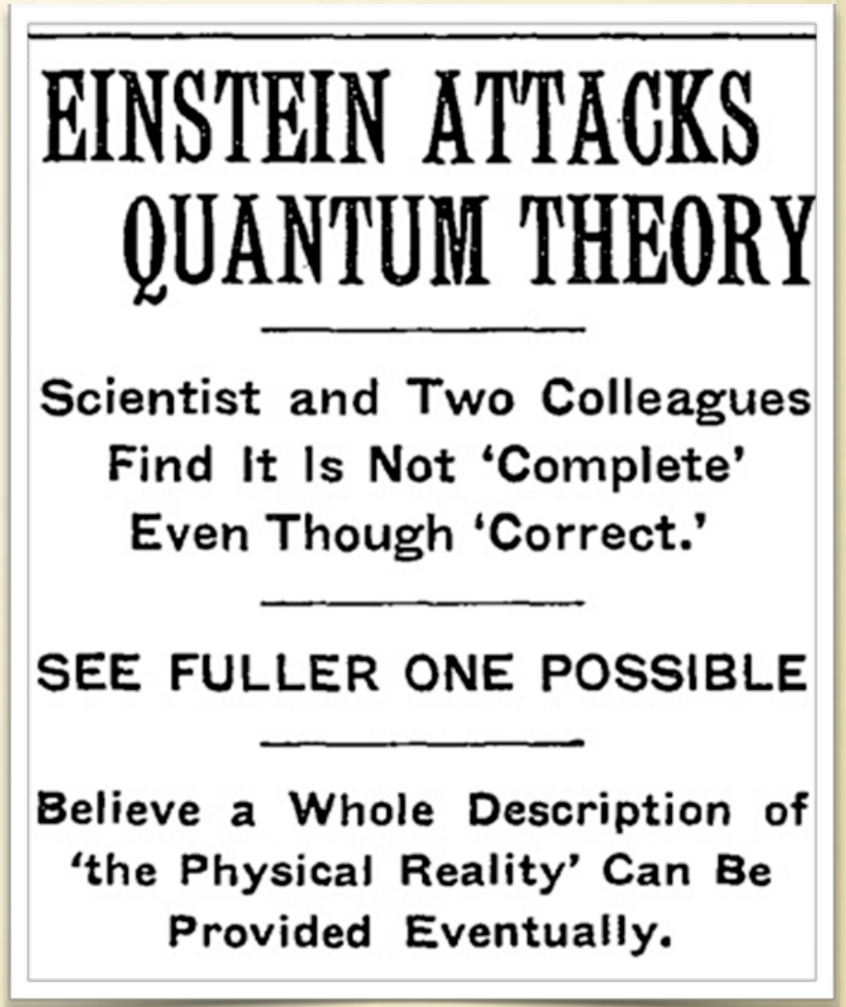

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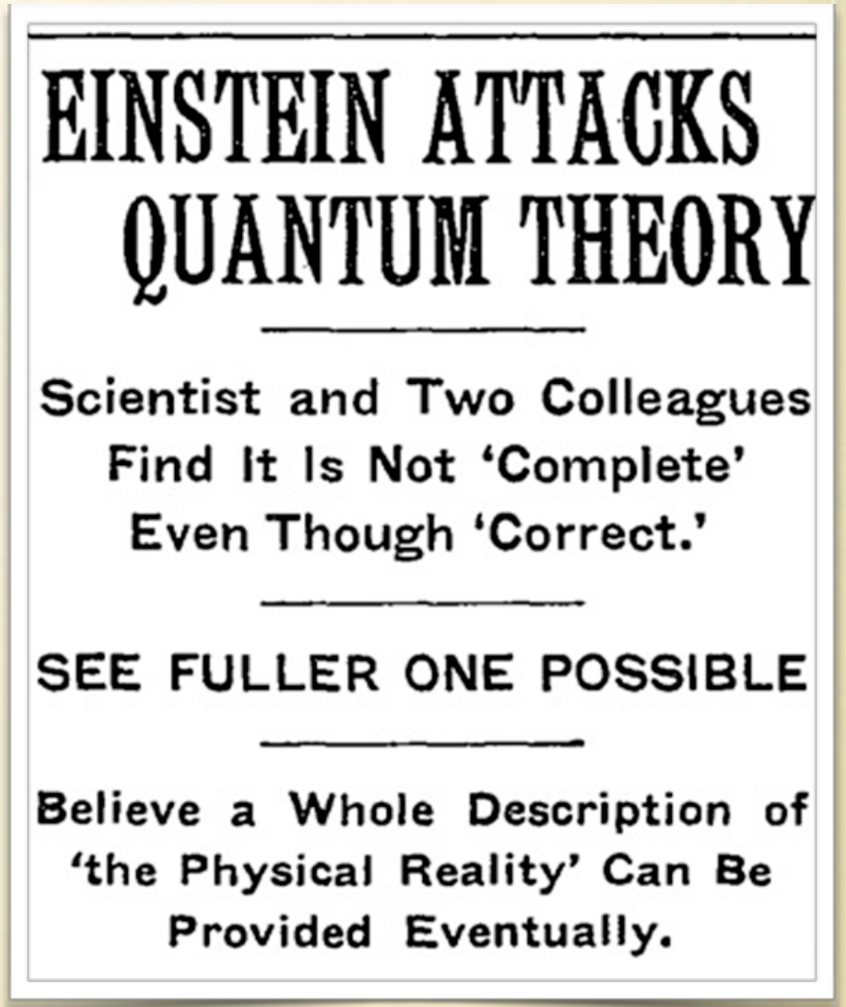

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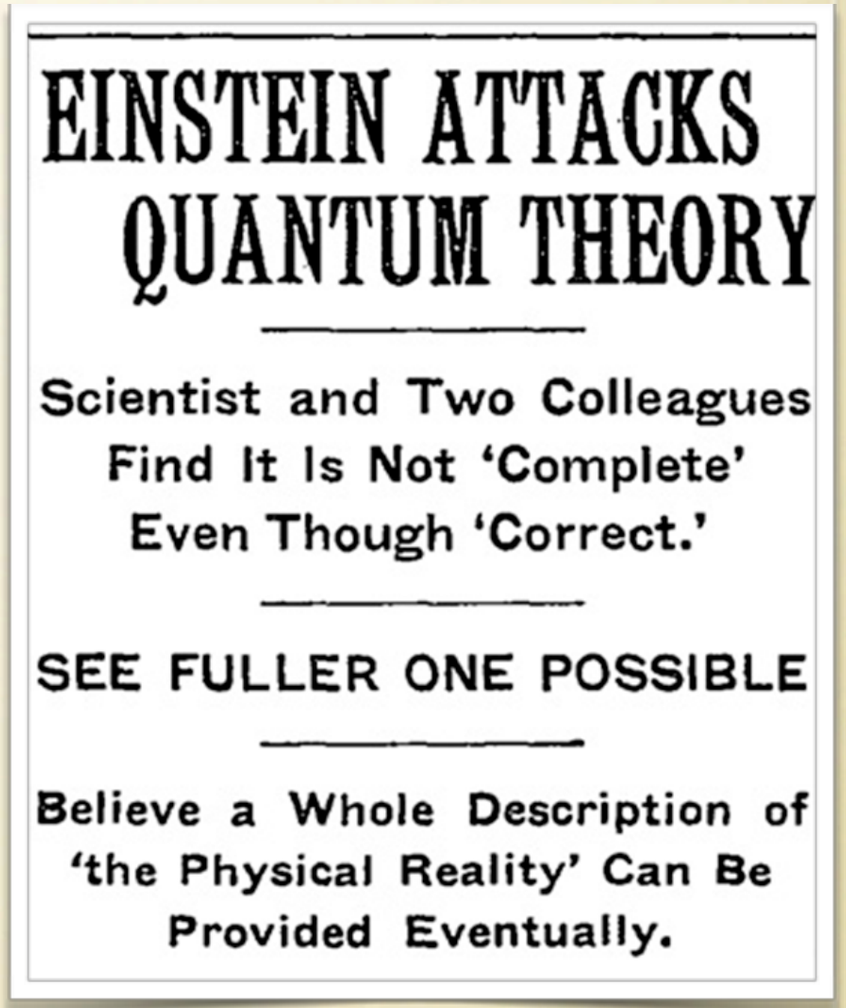

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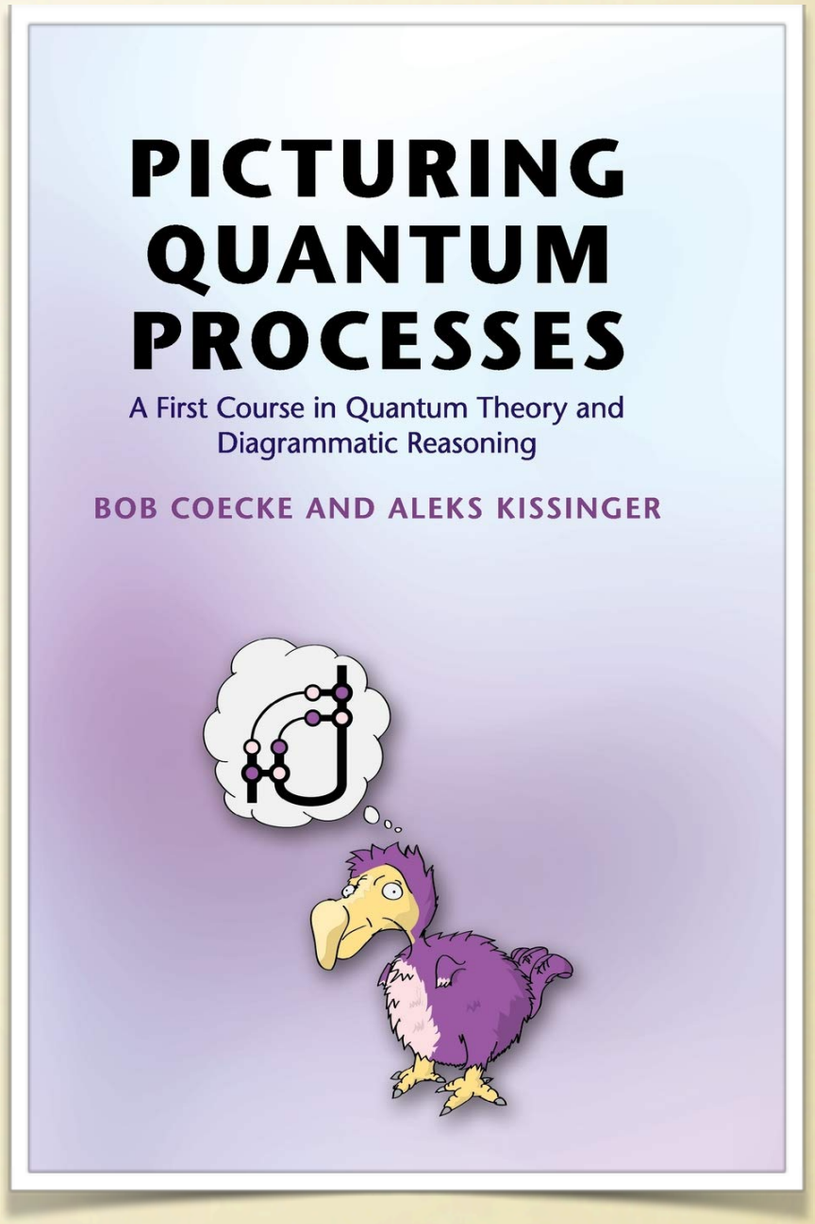
Why did it take so long?

Quantum Pictorialism



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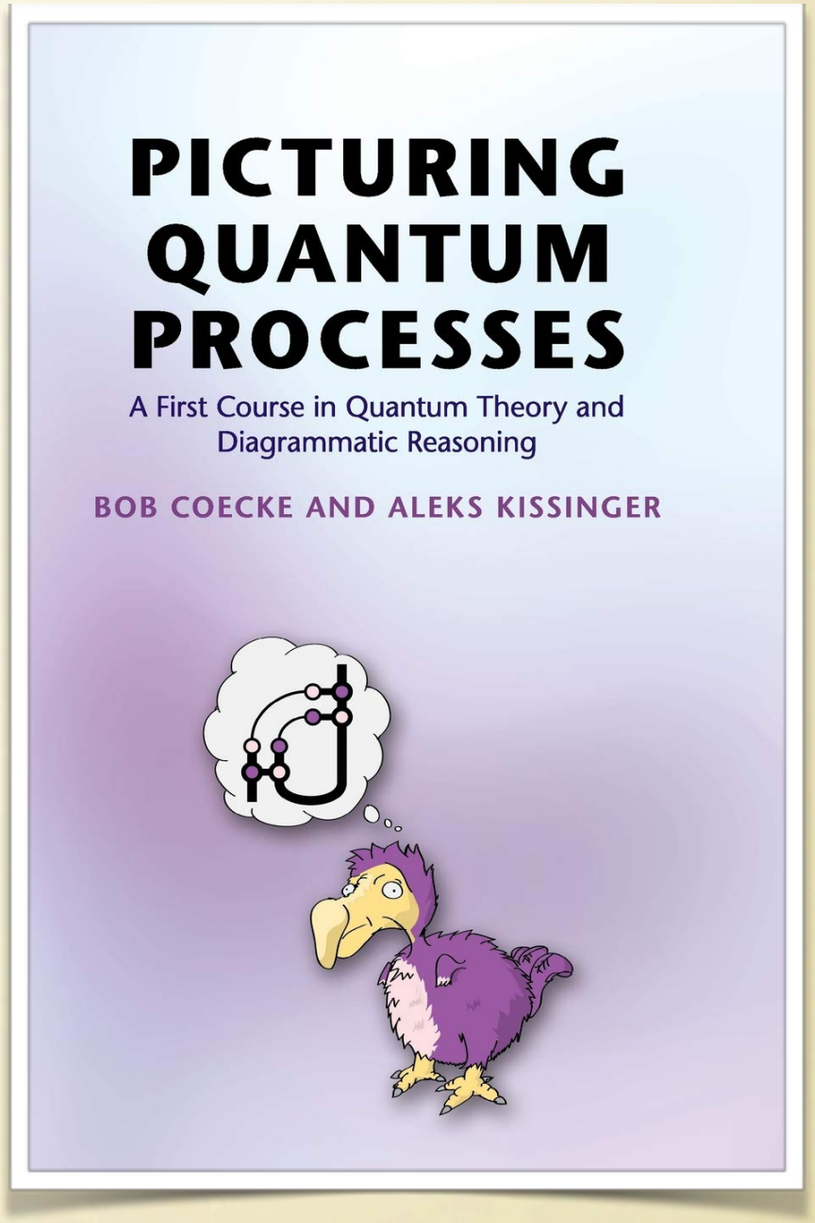
Reasoning about quantum systems via Hilbert spaces is rather incovenient, pretty much like programming a distributed application in Assembly



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$$\frac{1}{4}\begin{pmatrix} -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \end{pmatrix}$$

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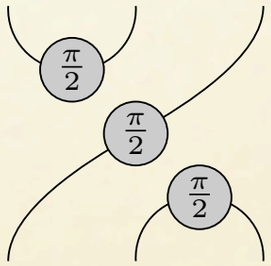

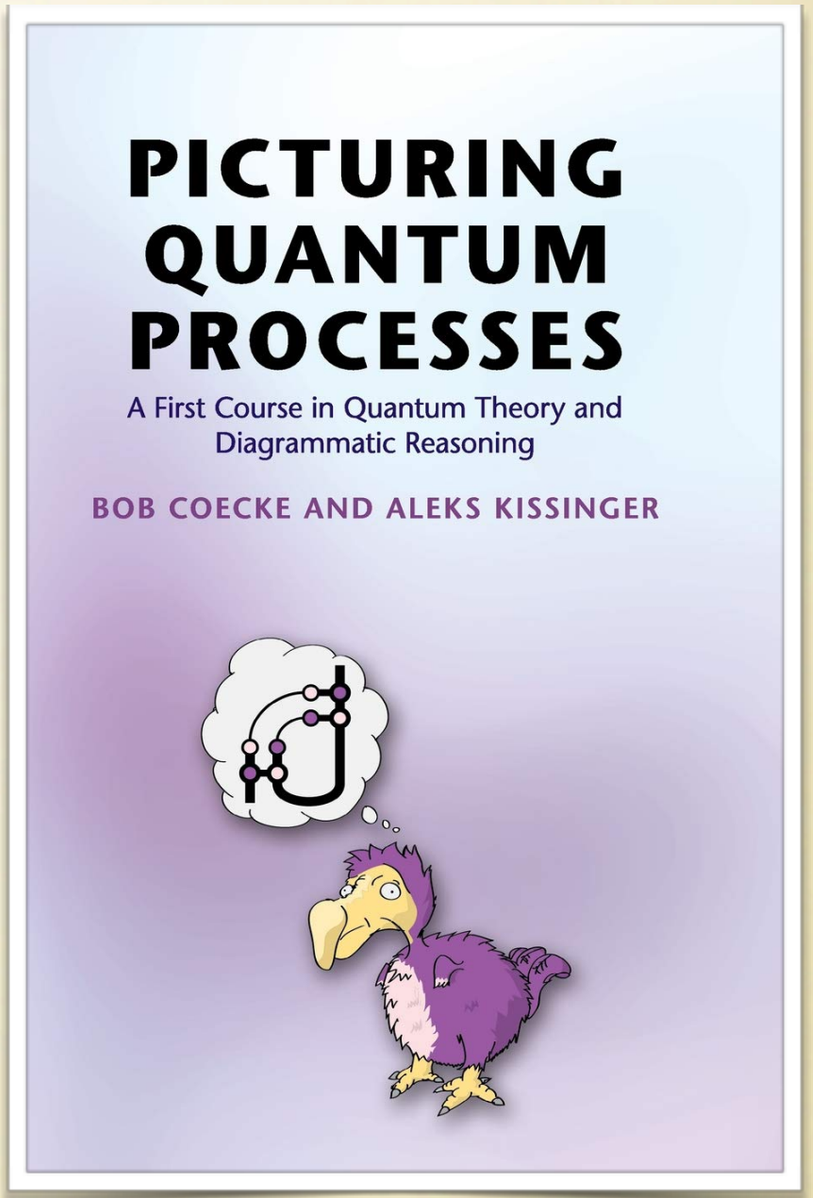

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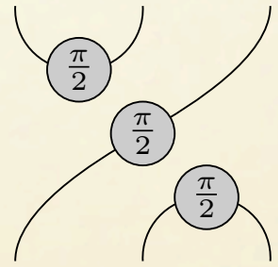

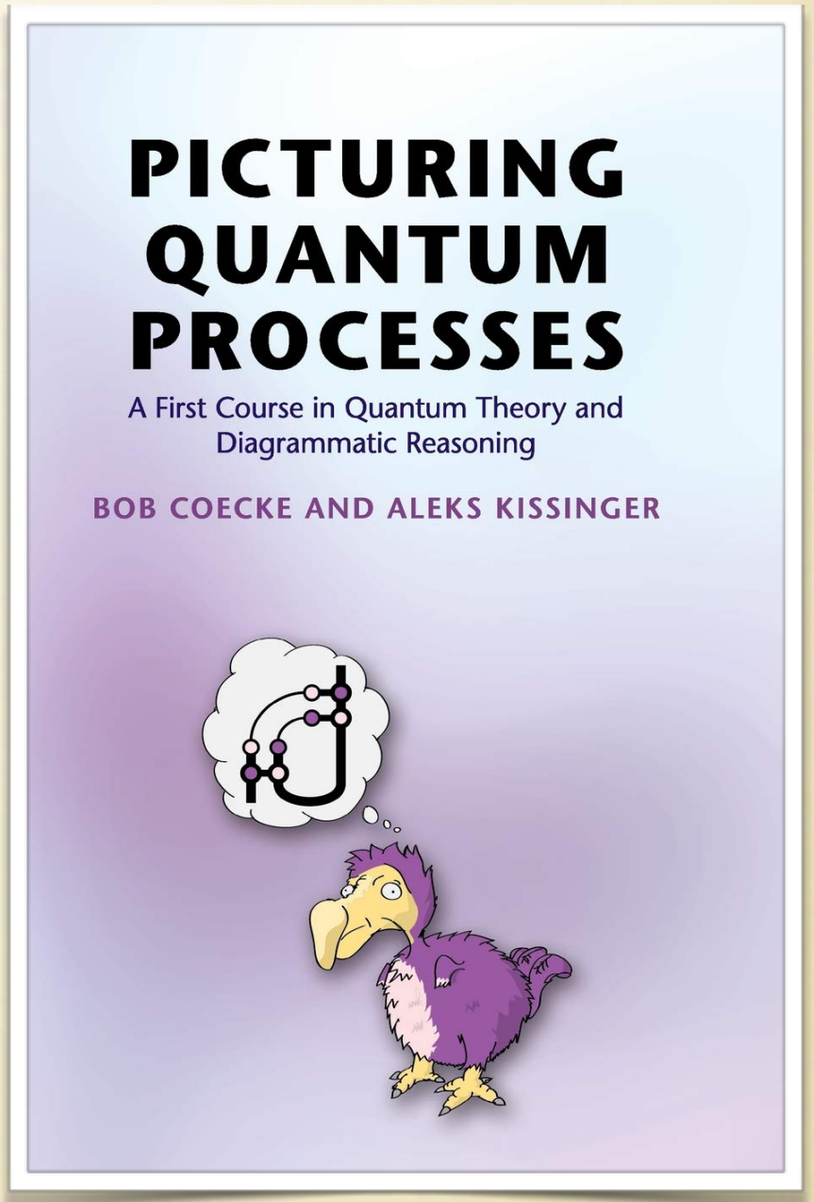

Developing an high level language for quantum system would boost the discovery of quantum features and the development of quantum technologies

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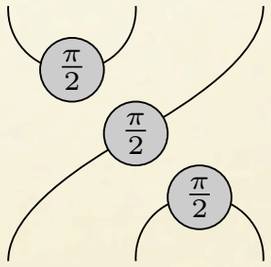

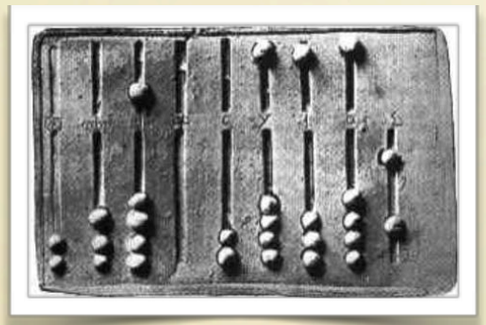

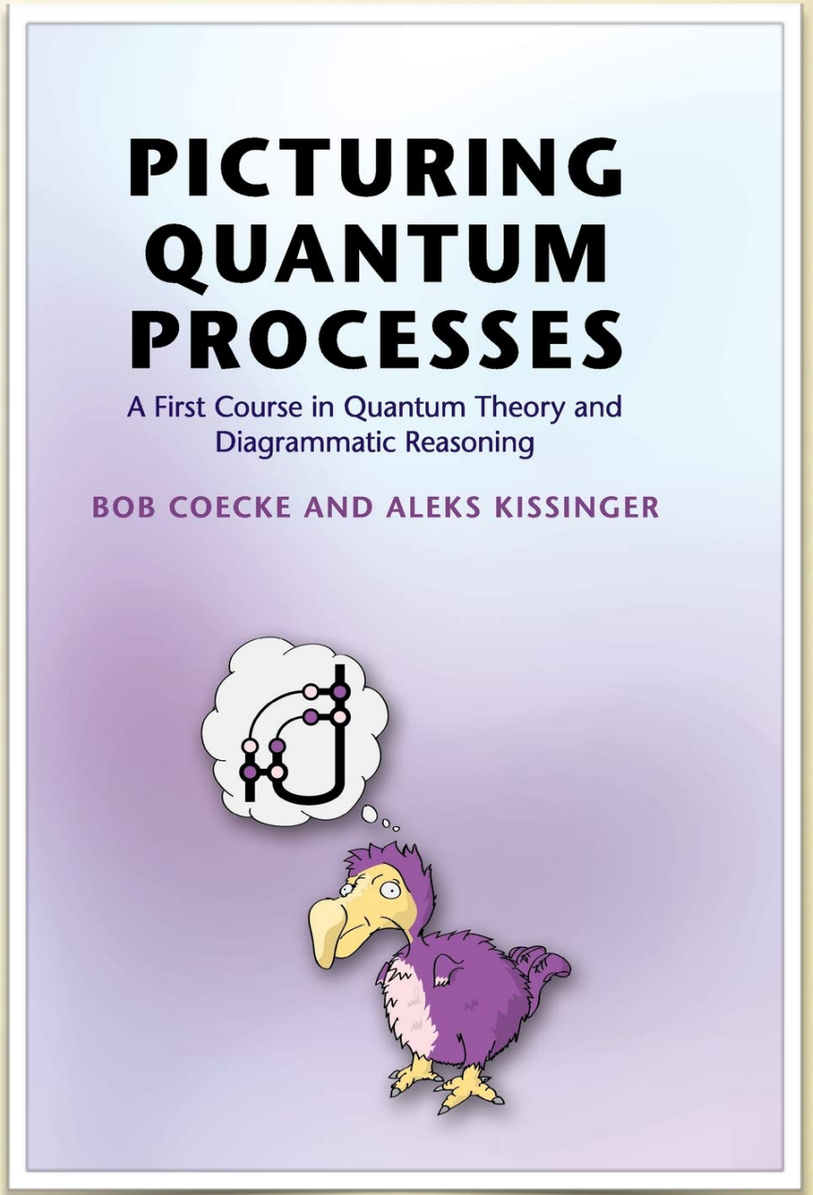

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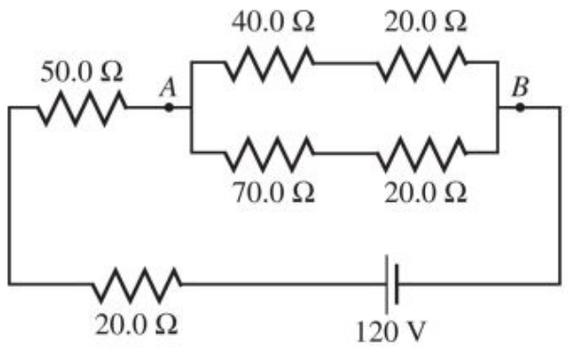
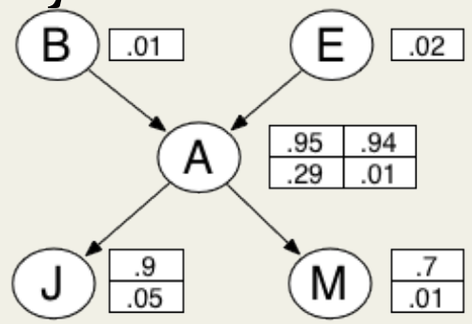
Network diagrams

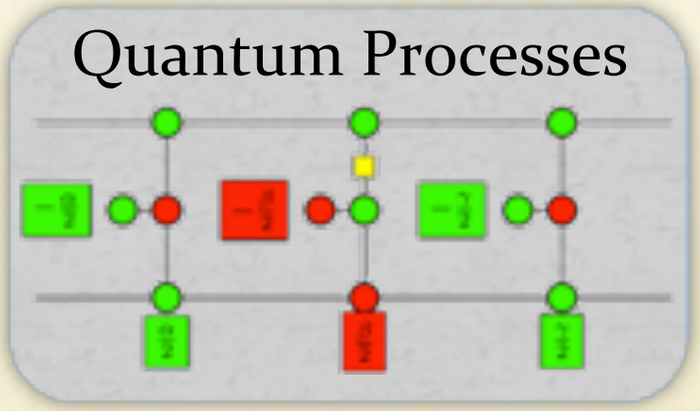
Electrical Circuits



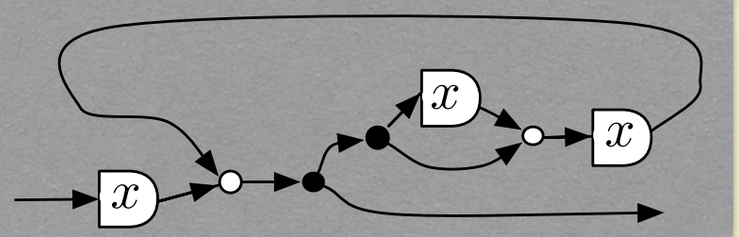
Bayesian Networks



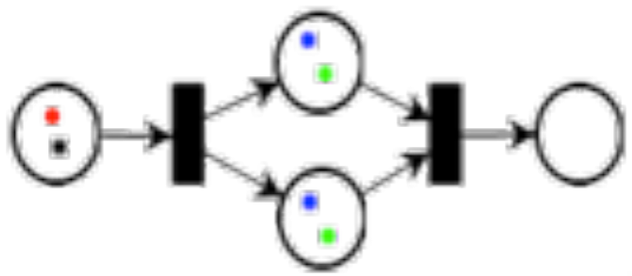
Quantum Processes



Signal Flow Graphs



Petri Nets



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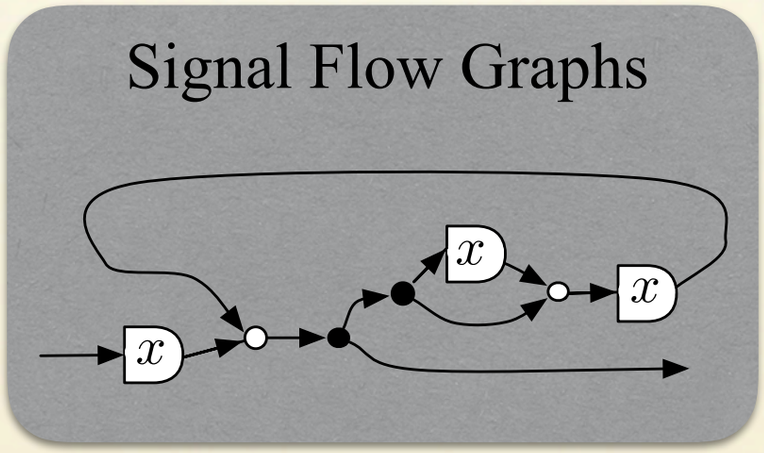

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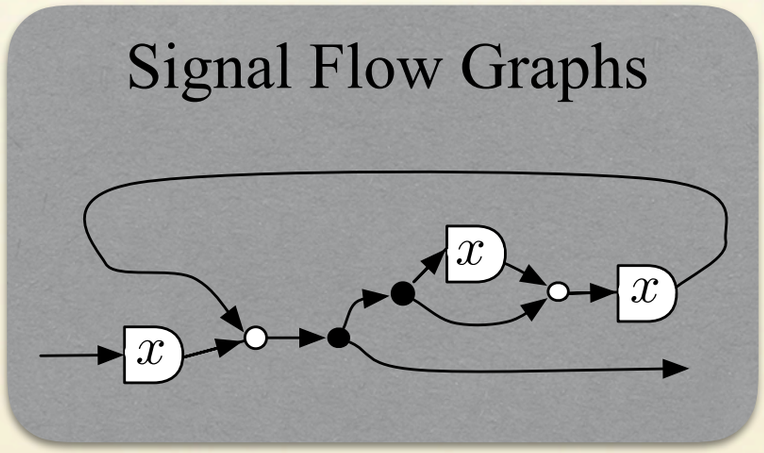

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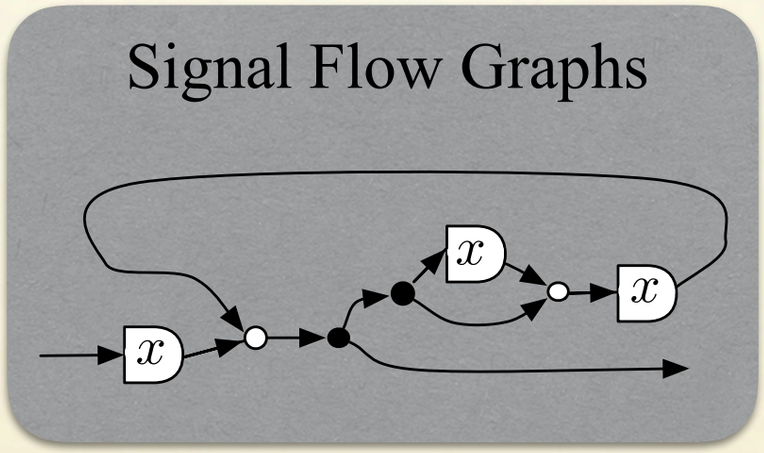

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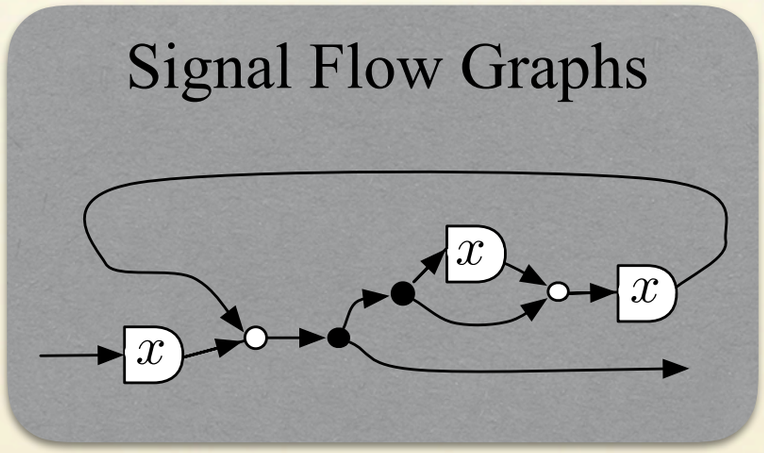

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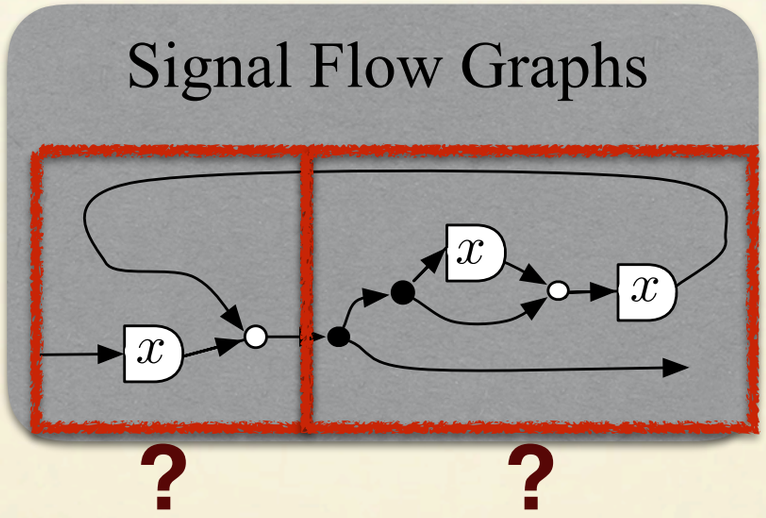

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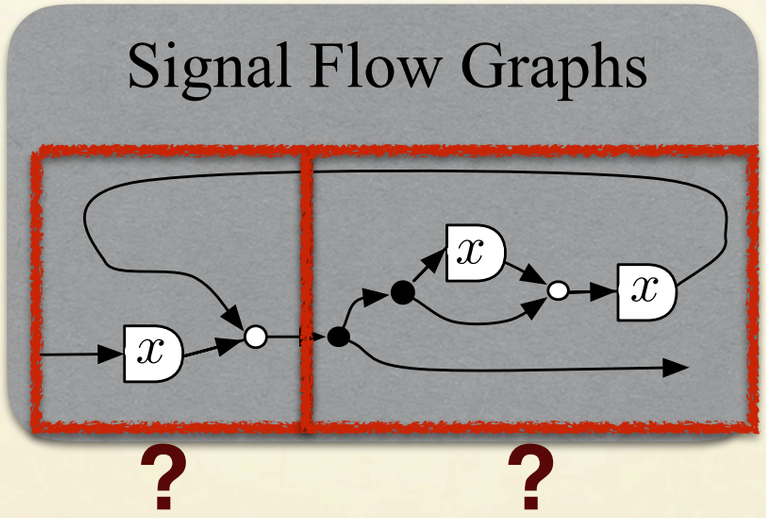

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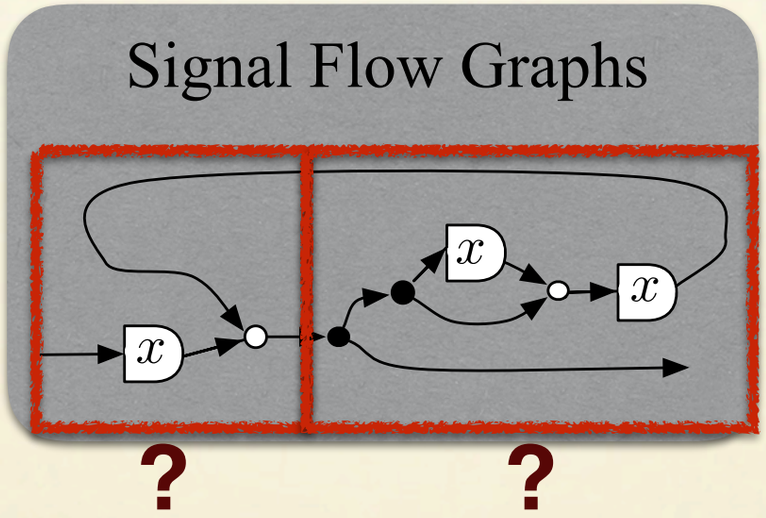

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https://www.azimuthproject.org/azimuth/show/Network+theory

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There is an emerging, multi-disciplinary field aiming at studying different sorts of networks **compositionally**, inspired by the **algebraic methods** of programming language semantics.

Diagrams are first-class citizens of the theory. The appropriate algebraic setting is **monoidal** (and not **cartesian**) categories.

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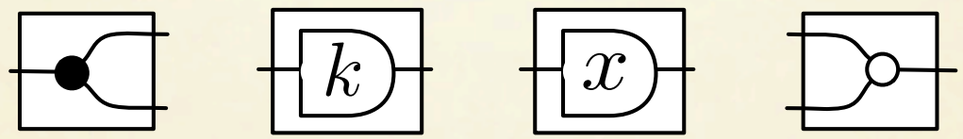

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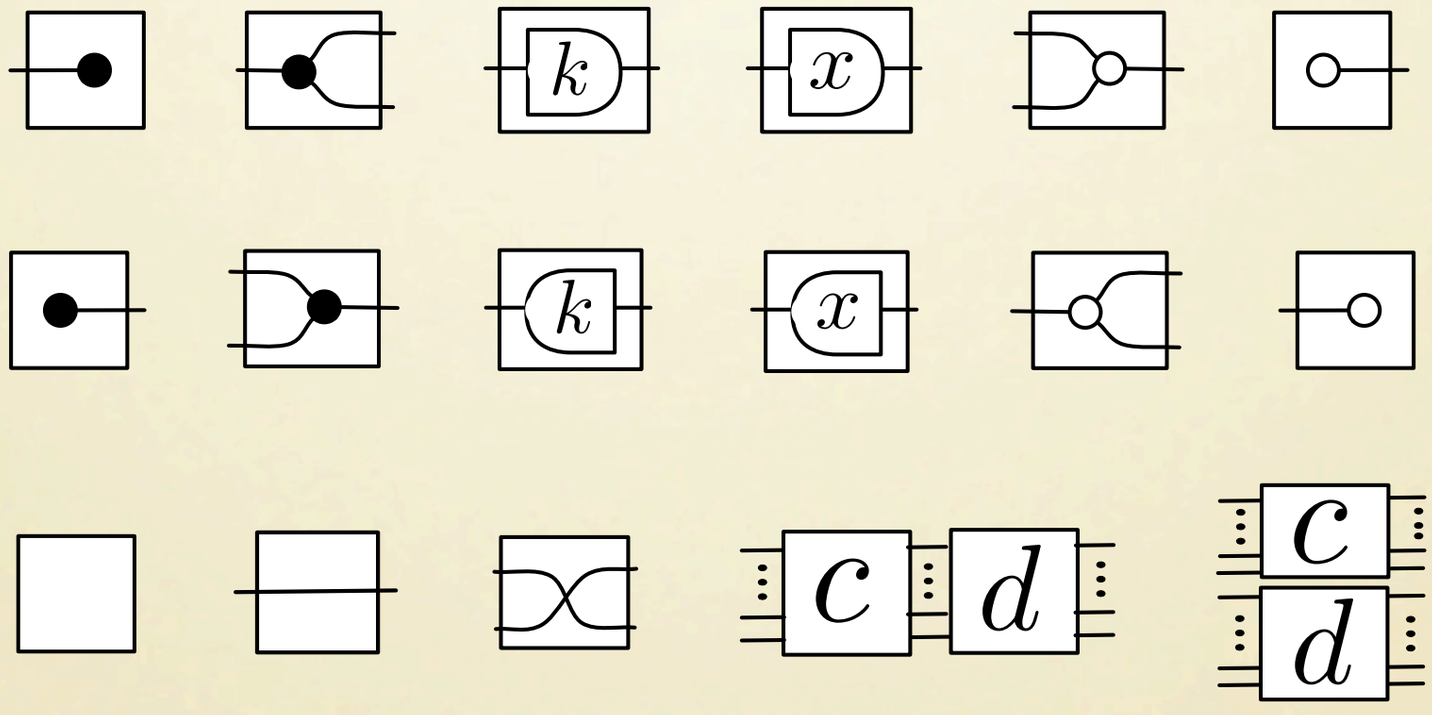
$c, d ::=$ 

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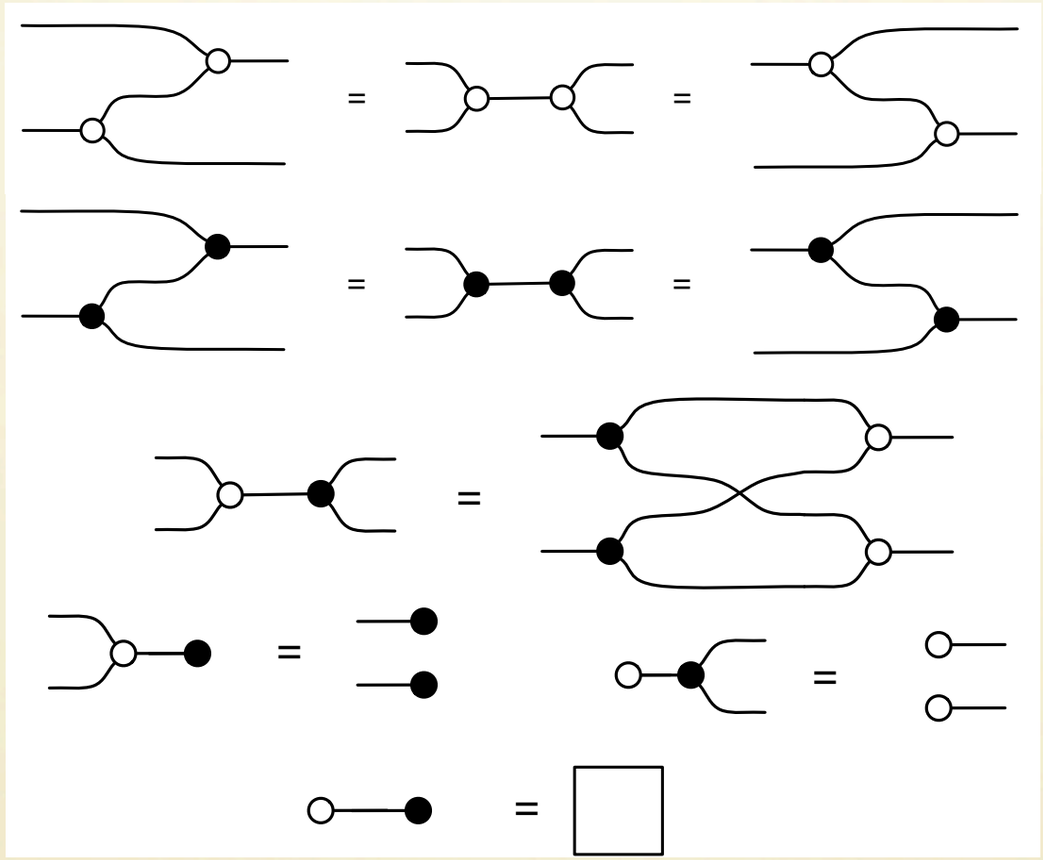
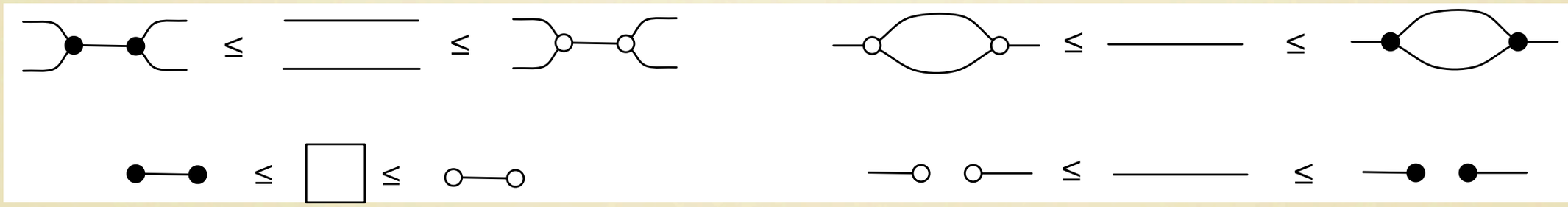
Sound and Complete Axiomatisation for Signal Flow Graphs





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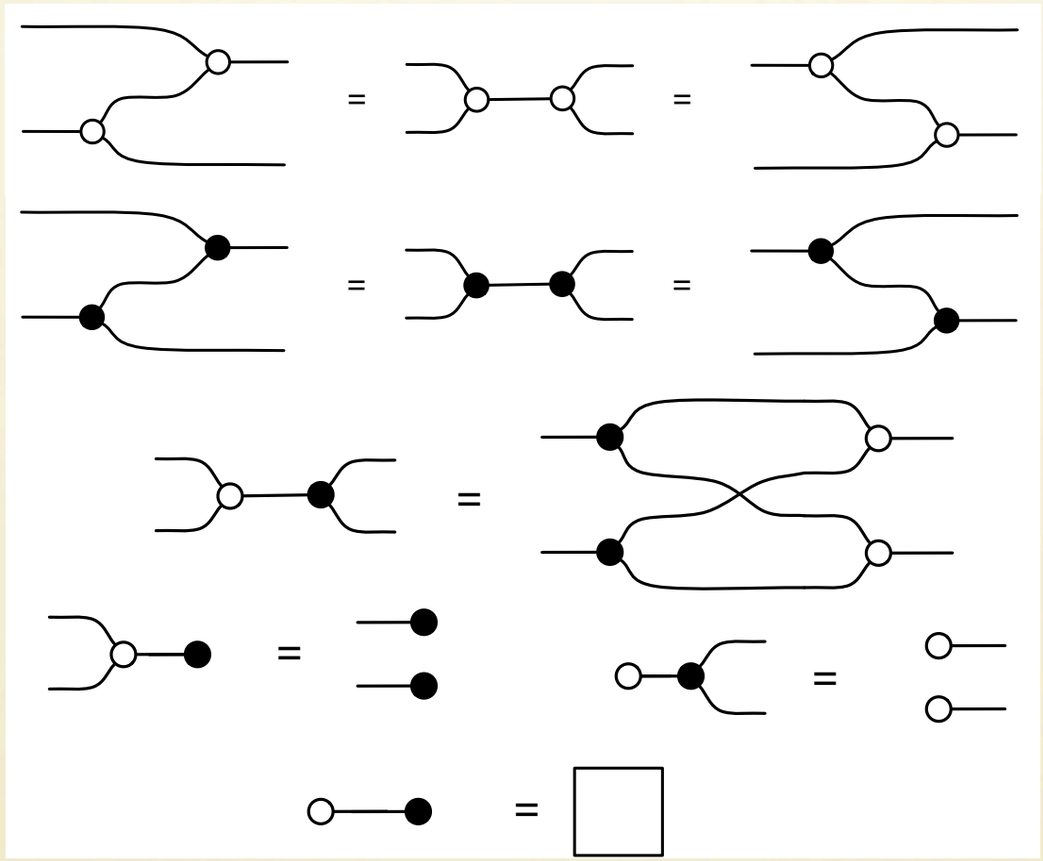

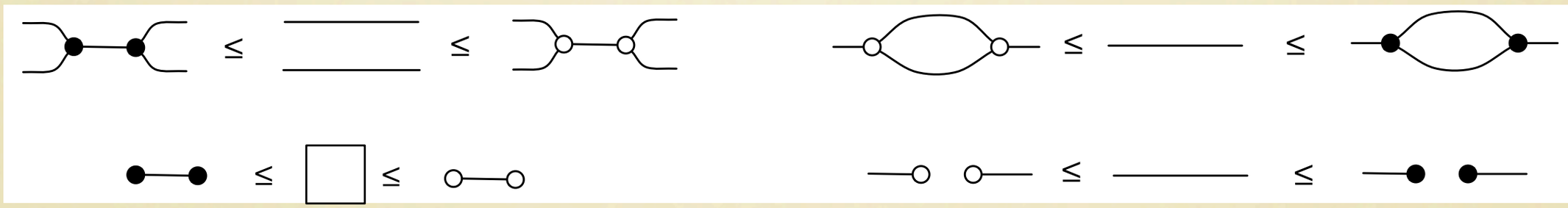

https://graphicallinearalgebra.net

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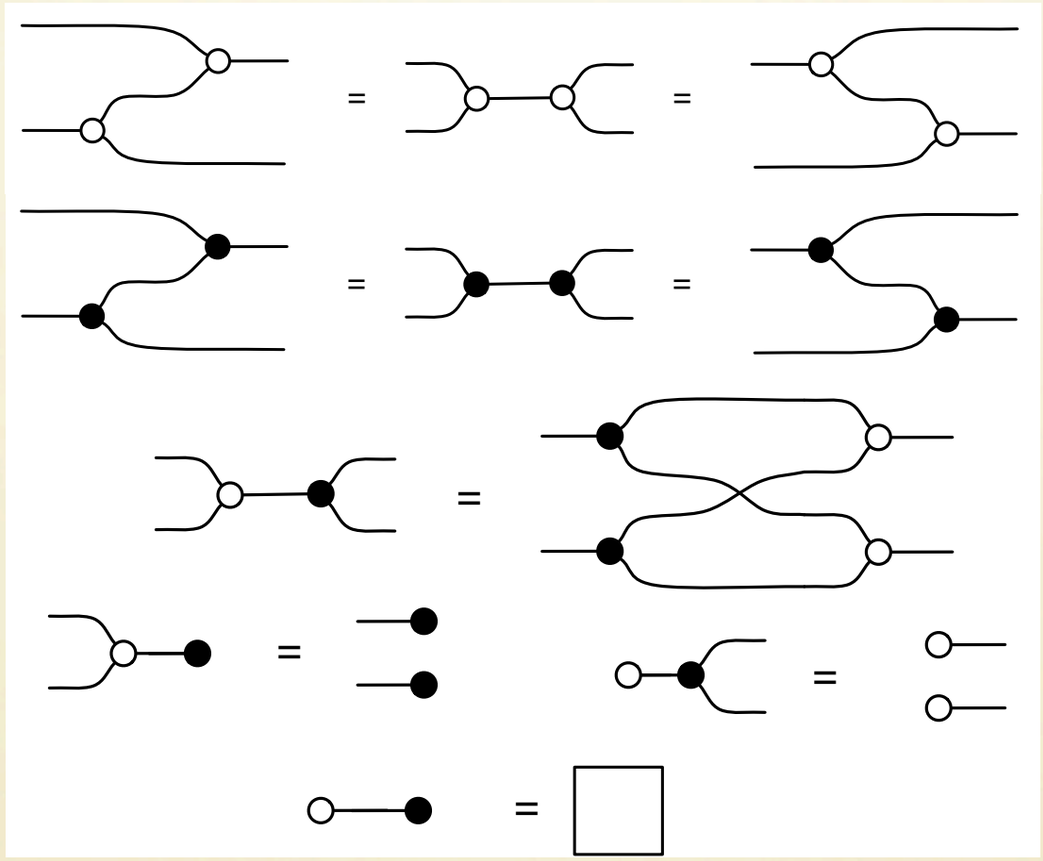
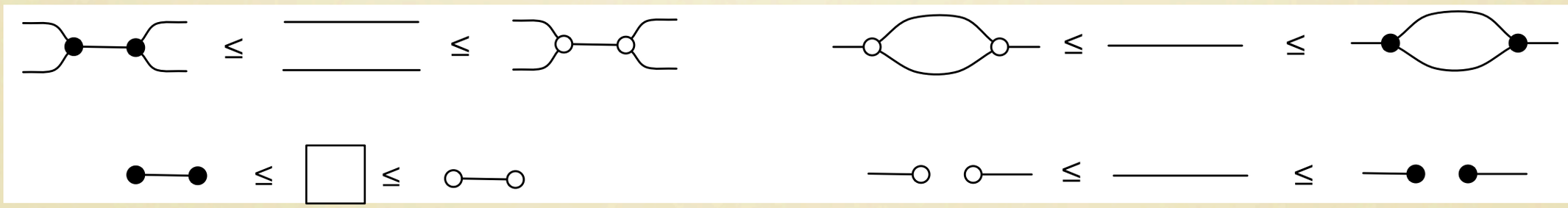
These axioms are almost the same as those for Quantum mechanics





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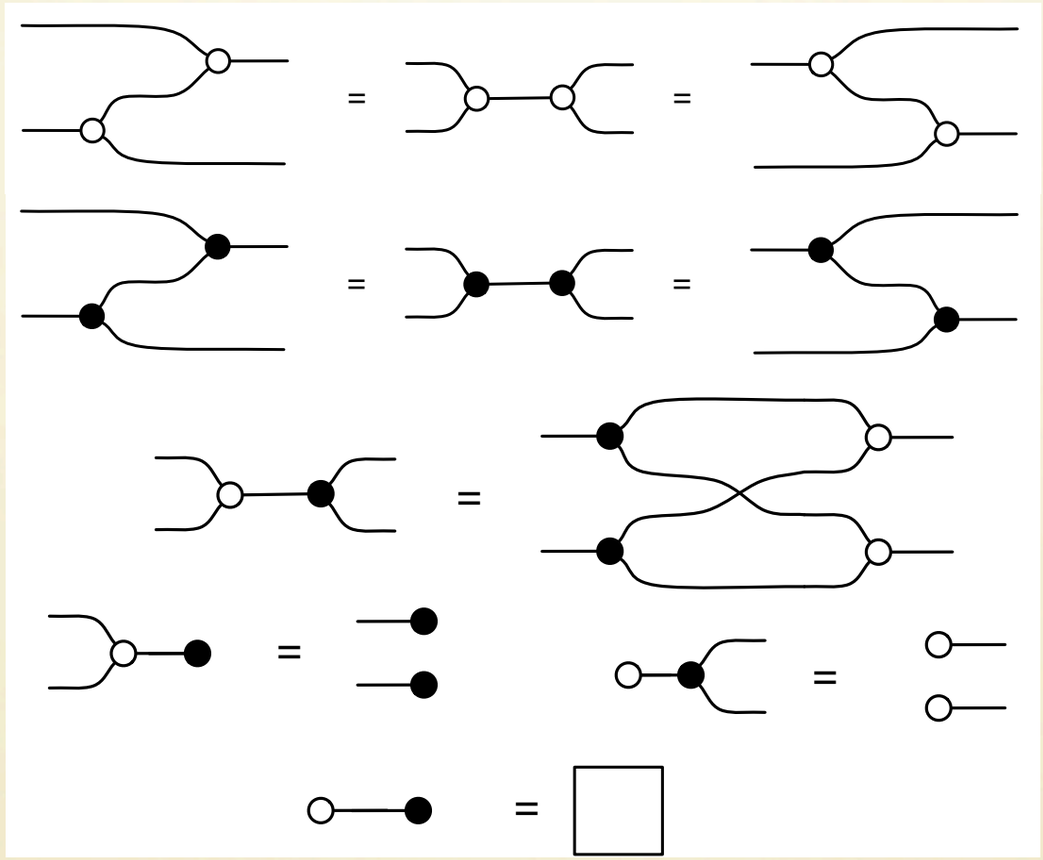
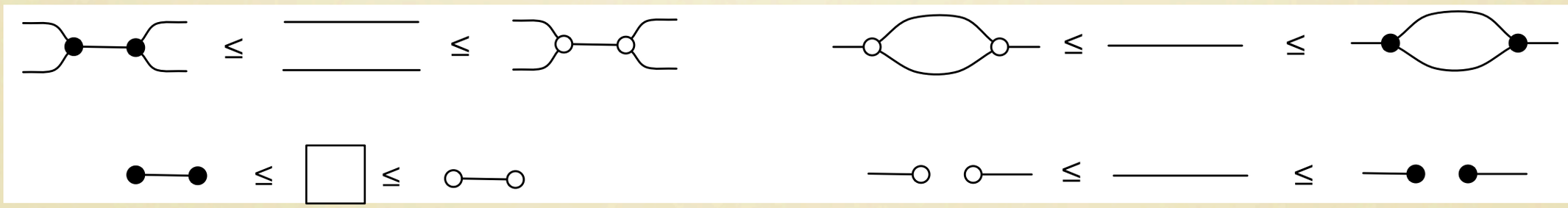


What is going on ?



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References

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